B4.1-R3: COMPUTER BASED STATISTICAL AND NUMERICAL TECHNIQUES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Compute

$$I = \int_0^1 [x/(x+10)]dx$$

using Simpson's rule of integration with h=0.2.

b) Let f(x) = ln x . Given the table of values

x:	0.40	0.50	0.70	0.80
ln x:	-0.916291	-0.693147	-0.356875	-0.223144

Estimate the value of ln (0.60).

c) An infinite sequence of independent trials is to be performed. Each trial results in a success with probability *p* and a failure with probability 1 – *p*. What is the probability that

i) at least 1 success occurs in the first *n* trials;
ii) exactly *k* successes occur in the first *n* trials;
iii) all trials result in successes?

d) Suppose that the joint density of X and Y is given by

$$f(x,y)=\begin{cases} exp[-(x+y)] & , 0\le x,y<\infty \\ 0 & , otherwise \end{cases}$$

Find E[X] and P(Y>1).

e) The average working-set size *X* of a program is normally distributed with unknown mean μ and a known variance $\sigma^2 = 81$. The program was executed 36 times and the average working-set size for each run recorded. The sample mean was computed to be 100 page frames. Assuming successive runs of the program are independent, find the 95 percent confidence interval for the mean average working-set size.

f) Write down probability density functions of

i) Uniform Variate
ii) Exponential Variate
iii) Normal Variate
iv) Gamma Variate

g) State the axioms of probability. Using these axioms, show that P(A)≤P(B) if $A \subseteq B$.

(7x4)

2.

a) Consider the set of equations:

$$\begin{aligned} 6x + 2y - z &= 4 \\ x + 5y + z &= 3 \\ 2x + y + 4z &= 27 \end{aligned}$$

Determine approximate solution of the system of equations using Gauss-Seidel method taking the initial solution as x=y=z=1.

b) For the data given below, write the Newton interpolating polynomial of degree 3 for $f(x) = x^2 e^{-x/2}$.

x:	1.10	2.00	3.50	5.00	7.10
f(x):	0.6981	1.4715	2.1287	2.0521	1.4480

Hence, find the error of the interpolate for *x*=1.75. (Given the tabulated value f(1.75) = 1.28611)

(9+9)

3.

a) In answering a question on a multiple choice test, an examinee either knows the answer or he guesses or he copies. Suppose each question has four choices. Let the probability that examinee copies the answer is 1/6 and the probability that he guesses is 1/3. The probability that his answer is correct given that he copied the answer is 1/8. Suppose an examinee answers a question correctly, what is the probability that he really knows the answer?

b) If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2, calculate

$$P(X=k \mid X+Y=n).$$

(8+10)

4.

a) The chances of three persons Mr. X, Y and Z becoming managers of a company are 4:2:3. The probabilities that bonus scheme shall be introduced if X, Y and Z become managers are 0.3, 0.5, 0.8 respectively. Find the probability that the bonus scheme will be introduced. What is the probability that X is appointed as the manager?

b) State central limit theorem. Using this theorem argue that if X is binomially distributed with parameters n and p, the distribution of $\frac{X - np}{\sqrt{npq}}$ approaches as standard normal Variate as $n \to \infty$.

c) If S_1^2 and S_2^2 are the standard deviations of independent random samples of size $n_1=61$ and $n_2=31$ from normal populations with $\sigma_1^2=12$ and $\sigma_2^2=18$, find $P((S_1^2 / S_2^2) > 1.16)$.

(6+6+6)

5.

a) Let X_i, i=1,…,10 , be independent random variables, each uniformly distributed over (0, 1). Calculate an approximation to $P(\Sigma X_i > 6)$.

b) If $X_1, X_2, ..., X_n$ is a random sample from a Normal population with mean μ and variance unity, then show that $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i^2$ is an unbiased estimator of $\mu^2 + 1$.

c) X is Poisson Variate with parameters λ. Obtain its Mgf. Hence find E[2X+3].

(6+6+6)

6.

a) Let X denote the main memory requirement of a job as a fraction of the total user-allocatable main memory of a computing center. The density function of X has the form:

$$f(x) = \begin{cases} k + 1)\, x^k & ,\text{if } 0 < x < 1,\ k > 0, \\ 0 & ,\text{ otherwise} \end{cases}$$

A large value of k implies a preponderance of large jobs. If k =0, the distribution-of-memory requirement is uniform. Suppose you have a sample of size 8 given by:

0.25, 0.45, 0.55, 0.75, 0.85, 0.85, 0.95, 0.90

Estimate the value of k, using the method of moments.

b) It is suspected that the number of errors discovered in a system program is Poisson distributed. The number of errors discovered in each one-week period is given as follows:

Number of errors in one-week period (i)	**Number of one-week period with i errors**	**Poisson probabilities**	**Expected frequencies**
0	14	0.150	7.50
1	11	0.284	14.20
2	9	0.270	13.50
3	6	0.171	8.55
4	5	0.081	4.05
5+	5	0.044	2.20

Assume that the total number of errors observed in the 50 weeks was 95. Estimate the rate parameter λ for the Poisson probabilities given above. Test the hypothesis that errors occur according to Poisson distribution using significance level of 0.05.

(8+10)

7.

a) Find the coefficient of linear correlation between the variables X and Y given in the table.

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

Also obtain the regression equation of Y on X for the data.

b) State the principle of least squares.

c) Show that cov(aX,Y) = a cov(X,Y).

(10+4+4)