B4.1-R3: COMPUTER BASED STATISTICAL AND NUMERICAL TECHNIQUES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ using trapezoidal rule. Take h=0.125.

b) Using Newton's forward interpolation formulae, find a cubic polynomial which takes the following set of values

x	0	1	2	3
$f(x)$	1	2	1	10

c) It is known that any item produced by a certain machine will be defective with probability 0.1, independently of any other item. What is the probability that in a sample of three items,
  i) none will be defective?
  ii) at most one will be defective?

d) Suppose $x_1, x_2, \ldots, x_n$ are independent and identically distributed with expected μ and variance σ^2. Defining $\overline{X} = \sum_{i=1}^{n} X_i / n$, obtain E[$\overline{X}$] and Var [$\overline{X}$].

e) The mfg of a random variate X is given by $M_X(t)=(0.8e^t+0.2)^{11}$.
  i) Find P(x>0)
  ii) Find $E[x^2]$

f) Let x_i, i=1, 2, …, 100 be independent random variables, each uniformly distributed over (0, 1). Using central limit theorem, obtain the probability

$$P\left(\sum_{i=1}^{100} x_i > 50\right).$$

g) Two file servers are compared according to their response time for retrieving a small file. The mean response time of 50 such requests submitted to server 1 was measured to be 682 ms with a known standard deviation of 25 ms. A similar measurement on server 2 resulted in a sample mean of 675 ms with a standard deviation of 28 ms. Test the hypothesis that server 2 provides better response than server 1 at 5% level of significance.

(7x4)

2.

a) Solve the system of equations

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

By using Gauss-elimination method.

b) Using the method of triangularisation, solve the following system of equations:

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2z = 20$$
$$5x + 2y + z = -12$$

(9+9)

3.

a) In a state lottery, a player must choose 8 of the numbers from 1 to 40. The lottery commission then performs an experiment that selects 8 of these 40 numbers. Assuming that the choice of the lottery commission is equally likely to be any of the $\binom{40}{8}$ combinations, what is the probability that a player has:

i) all 8 of the numbers selected;

ii) 7 of the numbers selected;

iii) at least 6 of the numbers selected?

b) An infinite sequence of independent trials is to be performed. Each trial results in a success will probability p and a failure will probability (1-p).

i) If we let x equal the number of trials performed until a success occurs, find E[X].

ii) Obtain the probability that exactly k failure occur in the first n trials.

(9+[6+3])

4.

a) The cumulative distribution function of the random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < 1 \\ \frac{2}{3} & 1 \le x < 2 \\ \frac{11}{12} & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

i) Draw the cdf of r.v.X.

ii) Compute P(x<3); P(x=1) and P(2<X≤4)

b) The joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{2} - ye^{-xy} & , \quad 0 < x < \infty, \; 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

i) Compute the conditional density of X given that Y=1 i.e. $f_{X|Y}(x|1)$

ii) Obtain $E[e^{X/2} | Y=1]$

(8+10)

5.

a) If X is a normal r.v. with parameters μ =3 and σ^2=9, obtain

i) P(2<X<5)

ii) P(X>0)

iii) P(|X-3|>6)

Express your results in terms of cdf Φ(z) of a standard normal variate Z~ N(0,1). The cdf Φ(z) is defined as $\Phi(z) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{z} exp[-y^2/2]\, dy$.

b) Buses arrive at a specified stop at 15-minute intervals starting at 7A.M. (i.e. they arrive at 7, 7:15, 7:30, 7:45 and so on), if a passenger arrives at a stop at a time that is uniformly distributed between 7 and 7:30 find the probability that he waits
   i) less than 5 minutes for a bus
   ii) more than 10 minutes for a bus.

(9+9)

6.

a) Define covariance between two random variables X and Y. What can you say about Cov(X, Y), when X and Y are independent?

b) Show that Cov(aX,Y) = a Cov(X,Y)

c) In 200 tosses of a coin, 115 heads and 85 tails were observed. Test the hypothesis that the coin is fair using significance level of
   i) 0.05, and
   ii) 0.01.

(5+6+7)

7.

a) In a sample of 600 men from a certain large city, 450 are found to be smoker. In another sample 900 from another large city, 450 are smoker. Do the data indicate that the cities are significantly different with respect to the prevalence of smoking among men?

b) The index of medical-care costs in United States for the years 1976-1984, taking the index for 1967 as 100 is given in the table:

Year	**Index of U.S. Medical-Care Costs (1967=100)**
1976	184.7
1977	202.4
1978	219.4
1979	239.7
1980	265.9
1981	294.5
1982	328.7
1983	357.3
1984	378.0

   i) Find the equation of a least-squares line fitting the data.
   ii) Estimate the index for the year 1985.

(8+10)