

B4.1-R3: COMPUTER BASED STATISTICAL AND NUMERICAL TECHNIQUES

NOTE:

1. Answer question 1 and any FOUR questions from 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.
 - a) The probability that a driver stopping at petrol station will have only his car tyre checked is 0.012, the probability that he will have the oil checked is 0.29 and the probability that he will have both oil and tyres checked is 0.07. What is the probability that a driver stopping at the station will have neither his car tyres nor oil checked?
 - b) Show that the variance of a binomial distribution cannot exceed $n/4$.
 - c) Let X , Y and Z be independent normal $(0, 1)$ random variables. Calculate $P(|X| < 1, |Y| < 2, |Z| < 3)$.
 - d) A function f has the values shown

x	1	1.25	1.50	1.75	2.0
f(x)	0.607	0.535	0.472	0.417	0.368

- Using Simpson's $\frac{1}{3}$ rule, approximate $\int_1^2 \exp[-\frac{x}{2}] dx$. When the function values at $x=1, 1.25, 1.50, 1.75$ and 2.0 are given. Compare with the true solution.
- e) Suppose we have a random sample of size $2n$ from a population denoted by X and $E\{X\} = \mu$ and $V(X) = \sigma^2$. Let

$$\bar{X}_1 = \frac{1}{2n} \sum_{i=1}^{2n} X_i \text{ and } \bar{X}_2 = \frac{1}{n} \sum_{i=1}^n X_i$$
 be two estimators of μ . Which is the better estimator of μ ? Explain your choice?
 - f) Assume that the time to failure T of a telephone switching system is exponentially distributed with a failure rate λ . Find the maximum likelihood estimate of the failure rate λ from a random sample $T_1=t_1, T_2=t_2, \dots, T_n=t_n$ of n times to failure.
 - g) Obtain a second degree polynomial approximation to

$$f(x) = (1+x)^{1/2}, \quad x \in [0, 0.01]$$
 Hence find $f(0.05)$.

(7x4)

2.
 - a) Solve the system of equations:

$$\begin{aligned} 2x + y + z &= 2 \\ x + 3y + 2z &= 2 \\ 3x + y + 2z &= 2 \end{aligned}$$
 by Triangularisation method.
 - b) Find a quadratic polynomial which takes the following values:

$$f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4$$
 Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum.

(9+9)

3.

- a) If 30 persons are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year?
- b) Suppose that a person chooses a letter at random from 'RESERVE' and then chooses one at random from 'VERTICAL'. Find the probability that the same letter is chosen.
- c) Five balls are randomly chosen, without replacement, from an urn that contains 5 red, 6 white and 7 blue balls. Find the probability that atleast one ball of each colour is chosen.

(6+6+6)

4.

- a) Write down pdf/pmf of the following
- i) Binomial Variate
 - ii) Poisson Variate
 - iii) Normal Variate
 - iv) Discrete Uniform Variate
 - v) Continuous Uniform Variate
- b) Suppose that X is a normal Variate with mean 5. If $P(X > 9) = 0.2$ approximately, what is Var X?

- c) The r.v. X has pdf $f(x) = \begin{cases} ax + bx^2 & , 0 < x < 1 \\ 0 & , otherwise \end{cases}$

If $E(X) = 0.6$, find

- i) $P(X < 0.5)$
- ii) Var X.

(5+5+8)

5.

- a) For the following probability distribution

x	-1	0	-1
y			
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

Find

- i) Marginal distribution of X and Y.
 - ii) The conditional distribution of X given $y = 2$.
- b) If X and Y are independent poisson variates with means λ_1 and λ_2 respectively, what is the probability that (i) $X + Y = k$, (ii) $X = Y$?

(9+9)

6.

- a) In a sample of 600 men from a certain large city, 450 are found to be smokers. In one of 900 from another large city 450 are smokers. Do the data indicate that the cities are significantly different with respect to the prevalence of smoking among men?

- b) A survey of 320 families with 5 children each revealed the following distribution.

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

Using χ^2 test, check whether the data is consistent with the null hypothesis that male and female are equally probable?

(9+9)

7.

- a) In partially destroyed laboratory record of an analysis of bivariate data (X, Y), the following results are legible: Var x = 9, Regression equations: $8x - 10y + 66 = 0$, $40x - 18y = 214$. Find
- The mean values of X and Y.
 - The correlation coefficient between X and Y.
 - The standard deviation of Y.
- b) The means of two single large samples 1000 and 2000 members are 67.5 inch and 68.0 inch respectively. Can the sample be regarded as drawn from the same population with standard deviation 2.5 inch?

(9+9)