B4.1-R3: COMPUTER BASED STATISTICAL AND NUMERICAL TECHNIQUES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If a box contains 75 good IC chips and 25 defective chips, and 12 chips are selected at random. Find

i) the probability that no chip is defective

ii) the probability that atleast one chip is defective.

b) A random variable X has pdf

$$f(x)=\begin{cases} Ax & , \quad 0 \le x < 5 \\ A(10-x) & , \quad 5 \le x < 10 \\ 0 & , \quad otherwise \end{cases}$$

i) Determine A

ii) Find $P(2.5 \le X \le 7.5)$

c) How many tosses of a fair coin are needed so that the probability of getting atleast one head is 0.875?

d) Using Lagrange's interpolation formula, find the function f(x) from the following table

X	0	3	4
f(x)	12	6	8

e) It is assumed that the arrival of the number of calls X per hour follows Poisson distribution with parameter λ. A random sample $X_1 = x_1$, $X_2 = x_2$, …, $X_n = x_n$ is taken. Obtain the maximum likelihood estimate of the average arrival rate.

f) For random variables X, Y and Z, it is given that

$Cov(X, Y) = \alpha$ and $Cov(X, Z) = \beta$

Obtain Cov(X, Y + Z).

g) Calculate the value of $\int_{-3}^{3} x^4 \, dx$ using Simpson's rule. Take seven equal subintervals.

(7x4)

2.

a) Consider the follow linear system of equations:

$10x_1 + x_2 + 2x_3 = 44$

$2x_1 + 10x_2 + x_3 = 51$

$x_1 + 2x_2 + 10x_3 = 61$

Using Gauss-Seidel method, compute x_1, x_2, x_3 after two iterations. Take the initial approximation as $x_1 = x_2 = x_3 = 0$.

b) Using the Newton algorithm, find the interpolating polynomial of least degree for the following table,

X	0	1	-1	2	-2
Y	-5	-3	-15	39	-9

(10+8)

3.

a) A system is made up of three components that operate independently from one another. For the system to function, atleast two components must function. We suppose that the probability of functioning of component no. 1 is equal to 0.95, that of component no. 2 is 0.9 and that of component no. 3 is 0.8.

i) What is the probability that the system functions?

ii) Given that the system functions, what is the probability that exactly two components function?

b) Two discrete random variables *X* and *Y* have joint pmf given by the following table:

		Y		
		1	2	3
X	1	1/12	1/6	C
	2	1/6	1/4	1/12
	3	1/12	1/12	0

i) Find C.

ii) Compute the probability of each of the following events.

- $X \leq 1\frac{1}{2}$.
- *X* is odd.
- *XY* is even.
- *Y* is odd given that *X* is odd.

(9+9)

4.

a) Out of every 100 jobs received at a computing center, 50 are of class 1, 30 of class 2 and 20 of class 3.

i) A sample of 30 jobs is taken without replacement. Find the probability that the sample will contain 10 jobs of each class.

ii) A sample of 30 jobs is taken with replacement, find the probability that there will be exactly 12 jobs of class 2.

b) A continuous random variable has the probability density function

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & , \ a < x < b \\ 0 & \text{otherwise} \end{cases}$$

i) Find the cumulative distribution function and sketch it.

ii) Find E[X] and Var X.

(9+9)

5.

a) A Binomial Variate X has the mean 6 and standard deviation $\sqrt{2}$. Find its moment-generating function. Hence obtain $E\left[e^{2X+3}\right]$. Also compute $E[X(X-1)]$.

b) If the probability that an individual suffers from a bad reaction from an injection of a given serum is 0.001, determine the probability that out of 2000 individuals (i) exactly 3 individuals will suffer a bad reaction, (ii) more than 2 individuals will suffer a bad reaction by using Poisson approximation. Also give exact probabilities (you need not simplify the expressions).

(9+9)

6.

a) The CPU time requirement X of a typical job can be modelled by the following distribution

$P(X \le t) = \alpha(1-e^{-\lambda_1 t}) + (1-\alpha)(1-e^{-\lambda_2 t})$, where α = 0.6, λ_1 = 10 and λ_2 = 1. Compute:

i) probability density function of X.

ii) the mean service time.

iii) Plot the distribution function and the density function of X.

b) State central limit theorem. If X is binomially distributed with parameters n and p, what can you say about the distribution of the variate $Y_n = \frac{X-np}{\sqrt{npq}}$ as $n \to \infty$?

c) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} exp[-(x+y)] & , \quad 0 \le x < \infty, 0 \le y < \infty \\ 0 & , \quad otherwise \end{cases}$$

Compute E[X] and P[Y>2].

(6+6+6)

7.

a) The failure rate of a certain electronic device is suspected to increase linearly with its temperature. Fit a least-squares linear line through the data (two measurements were taken for each given temperature)

Temp ° F	55	65	70	85	95	105
Failure rate $.10^6$	1.90	1.93	1.97	2.00	2.01	2.01
	1.94	1.95	1.97	2.02	2.02	2.04

i) Obtain desired least-squares line.

ii) Obtain predicted failure rate at 70°F.

b) A computer system has 6 I/O channels and the system personnel are reasonably certain that the load on channels is balanced. If X is the random variable denoting the index of the channel to which a given I/O operation is directed, that its pmf is assumed to be uniformly distributed. Out of N=150 I/O operations observed, the number of operations directed to various channels were:

n_0=22, n_1=23, n_2=29, n_3=31, n_4=26, n_5=19.

Test the hypothesis that the load on the channels is balanced at 5 percent level of significance.

(9+9)